Experiment on Falling Balls through a Transparent Viscous Liquid

Masaru SHODA

(Received September 10, 1968)

To measure the viscosity of a transparent viscous liquid such as glycerine, the falling ball method based on the Stokes's law is widely used. The essential part of this method is the measurement of velocity of falling ball through liquid, which is influenced by the proximity of the walls of the containing vessel, as well as by the radii of balls. This paper describes the results of measurement of falling velocities of balls through the liquid (glycerine) under various conditions. The illustlated example of viscosimetric measurement of glycerine is given. By means of the rotating shutter strobe camera, the initial transitional phenomena of motion of falling balls through liquid until a terminal velocity is reached are also studied.

INTRODUCTION

A variety of methods[1], are available for the measurement of viscosity of liquid ; for example, the methods by observation of the flow of the liquid through a tube, the coaxial cylinder method and the oscillating disk method are used. For the more viscous liquid, however, it is convinient to use the falling ball method by the Stokes's law. The measurement of velocity of falling balls are the essential part of the method, and it depends sensitively upon the radius of ball, the size of containing vessel and tempe rature of liquid. The chief object of this experiment is to study the velocity change of falling balls under various conditions and to see the transitional features of falling balls through the liquid.

I. EXPERIMENTALS

According to the Stokes's law, the spheres move with a terminal velocty v_0, which is given by

$$F=6\pi\eta v_0 r$$

where F is the viscous force acting on the sphere of radius r cms. and η is the viscosity.

In the steady state F is equal to the net downward force

i. e. $$6\pi\eta v_0 r=\frac{4}{3}\pi r^3(\rho-\delta)g$$

i. e. $$v_0=\frac{2}{9}\frac{r^2}{\eta}(\rho-\delta)g \cdots\cdots\cdots\cdots(1)$$

where ρ is the density of the steel ball and δ is the density of the liquid (glycerine).

As the velocity of fall is influenced by the proximity of the walls of the containing vessel, thes pheres should therfore be dropped centrally in the vessel, and corrections for size effects should be considered in the experiments.

A relation between v the observed terminal velocity in a vessel of radius R, and v_0, the velocity in a vessel of infinite radius has been given, viz.,

$$v_0 = v(1+2.4)\frac{r}{R} \cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

This is the equation for correction.

Now a glass tube ($\phi=12.2$ mms.) is filled with the liquid (glycerine 85~86%), whose temperature is 30°C.

The ball bearing ($\phi=1.2$ mms., 0.007192 gms.) is transferred to the glass tube, and after dropping a few cms. their terminal velocity is obtained by timing a fall. To avoid parallax errors paper collars are attached to the glss tube, and the upper edge of these collars are used as points of reference for the determination. The distance between two points of reference is 64.4 cms.

The time of falls is given as follows:

n	sec.	$\Delta\cdot 10$	$\Delta^2\cdot 10^2$
1	108.4	4	16
2	8.4	4	16
3	8.2	2	4
4	8.0	0	0
5	7.0	10	100
6	8.0	0	0
mean	108.0		$\Sigma\Delta^2=136$

$$\text{mean error} = \pm\sqrt{\frac{\Sigma\Delta^2}{n(n-1)}} = \pm\sqrt{\frac{136\times 10^{-2}}{6\times 5}} = \pm 0.22$$

$$\text{relative mean error} = \pm\frac{0.22}{108.0}\times 100\% = \pm 0.2\%$$

The apparent terminal veloity v is given :

$$v = \frac{64.4}{108.0} = 0.596 \text{ cm./sec.}$$

Since the velocity v is given as $v=\frac{l}{t}$, the maximum relative mean error is given as $\frac{\Delta v}{v}=\frac{\Delta l}{l}+\frac{\Delta t}{t}$, where l is the distance between points of reference on the tube.

Put the values $l=64.4$ cms., $\Delta l=\pm 0.05$ cms. (estimated) and $\frac{\Delta t}{t}=\pm 0.2\%$, then we obtain the value $\frac{\Delta v}{v}=\pm 0.3\%$.

Using the Eq. (2), we may calculate the v_0 :

$$\underline{v_0 = 0.736 \text{ cm./sec.} \quad \pm 0.3\%}$$

II. VERIFICATION OF Eq. (2)

Using the tubes of different radius, Eq. (2) may be tested. The ballbearings ($\phi=$ 1.2 mm.) are dropped into the tubes of different radius filled with glycerine, (1.26 gm./c. cm.). The temperature is maintained at 28 °C. All the measurements were carried out under the same conditions. The results of the experiment are shown in Table I.

Table I Expeirmental results.

Tudes	I	II	III	IV	V
Diameter of tube 2R mm.	25.3	21.9	17.8	15.6	12.2
Ball bearing used $\phi = 1.2$ mm.	〃	〃	〃	〃	〃
Apparent terminal velocity v cm. /sec.	0.706	0.677	0.664	0.645	0.596
Corrected terminal velocity v_0 cm. /sec.	0.788	0.767	0.772	0.754	0.736

The terminal velocities in different tubes are plotted in Fig. 1.

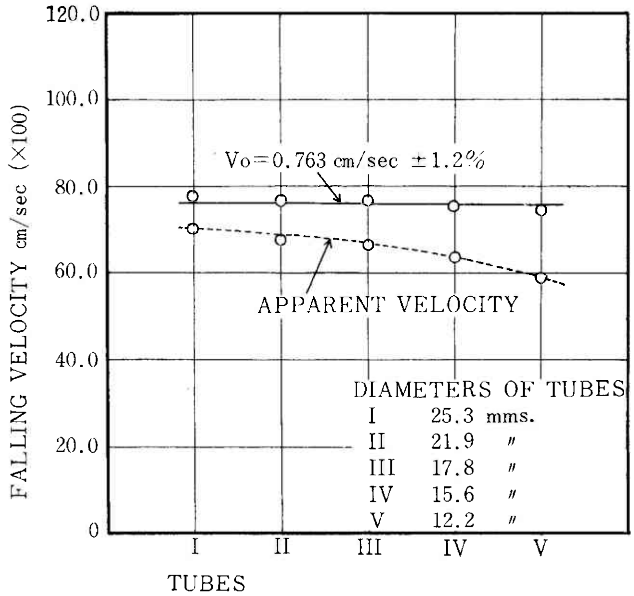

Fig. 1. The falling vecloities in different tubes are plotted.

As shown in Fig. 1 the corrected value of terminal velocity, i. e. the velocity in a vessel of infinite radius can be found as 0.763 cm. /sec. ± 1.2%.

The measurements of terminal velocity through liquid (glycerine) with ball bearings of different radius were performed. Experimental results are shown in Table II. Each ball bearing used consists of same material.

Table II Experimental results.

Diameter of ball bearing 2r mm.	0.5	1.0	1.2	1.5
Diameter of tube 12.2 mms.	〃	〃	〃	〃
r^2 mm^2.	0.0625	0.2500	0.3600	0.5625
Apparent terminal velocity v cm. /sec.	0.1095	0.402	0.567	0.796
Corrected terminal velocity v_0 cm. /sec.	0.1190	0.470	0.686	1.030

The experimental plot of terminal velocity against square of radius of ball bearing is illustrated in Fig. 2.

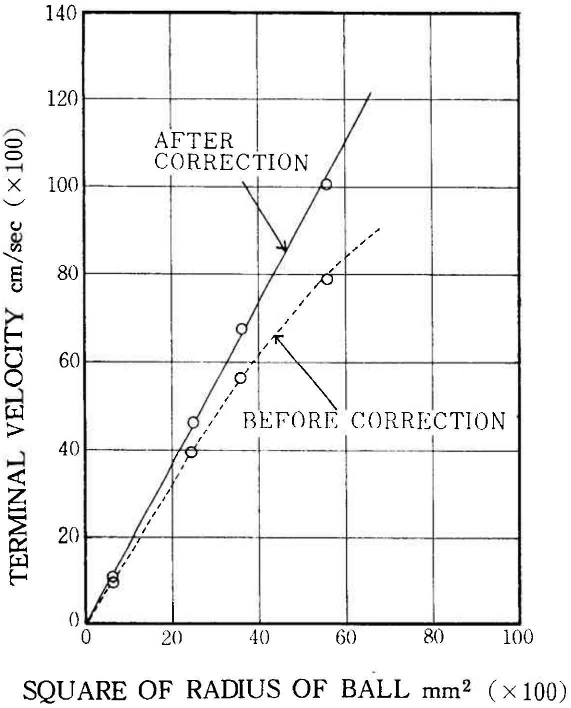

Fig. 2 The plot of terminal velocity against square of radius of the ball.

Fig. 2 shows the good lineality (shown as a solid line) between the corrected terminal velocity and square of radius of ball, which is the verification for Eq. (1) as well as Eq. (2).

III. EXAMPIE

We now give an example of a measuement of the viscosity of glycerine by falling ball method. The data are given as follows :

ball bearing ($2r=1.2$ mm., $\rho=7.951$ gm. /c. cm.)(given)
glss tube ($2R=17.8$ mms.) (measured)
distance betw. points of reference on the tube
($l=68.4$ mms.) (measured)
glycerine ($\delta=1.259$ gm. /c. cm.) (measured)
temperature (29 °C) (measured).

The data of measurement of falling time between. reference points by a stop watch are given as follows :

n	sec.	$\Delta \cdot 10$	$\Delta^2 \cdot 10^2$
1	93.0	− 5	25
2	4.0	+ 5	25
3	2.6	− 9	81
4	3.0	− 5	25
5	3.8	+ 3	9
6	3.6	+ 1	1
7	4.0	+ 5	25
8	3.4	− 1	1
9	4.0	+ 5	25
10	3.4	+ 1	1
mean	93.5		$\Sigma\Delta^2=218$

mean error $\pm\sqrt{\dfrac{\Sigma\ \Delta^2}{n(n-1)}}=\pm\sqrt{\dfrac{218\times10^{-2}}{10\times9}}=\pm\ 1.56$ sec.

relative mean error $\pm\dfrac{1.56}{93.5}\times100\%=0.17\%.$

We can calculate the apparent terminal velocity of falling ball as follows:

$$v=\frac{l}{t}=\frac{68.4}{93.5}=0.732 \text{ cm./sec.}$$

Hence the terminal velocity corrected for size effect is given:

$$v_0=v\left(1+2.4\frac{r}{R}\right)=0.836 \text{ cm./sec.}$$

Thus by putting these data into Eq. (1),

i. e. $\eta=\dfrac{2r^2(\rho-\delta)}{9\ v_0,}.$

we obtain the value of viscosity ;

$\eta\ =\ 6.28$ P at 29°C.

Let us calculate the relative mean error of results of our measurement. From Eq. (1), neglecting the error due to Eq. (2),

$$\frac{\Delta\eta}{\eta}=\frac{\Delta(\rho-\delta)}{\rho-\delta}+2\frac{\Delta r}{r}+\frac{\Delta v}{v}$$

where $\dfrac{\Delta v}{v}$ is substituted by $\dfrac{\Delta l}{l}+\dfrac{\Delta t}{t}$ and therefore,

$$\frac{\Delta\eta}{\eta}=\frac{\Delta\rho-\Delta\delta}{\rho-\delta}+2\frac{\Delta r}{r}+\frac{\Delta l}{l}+\frac{\Delta t}{t}$$

Additional data are given :

$\Delta\delta=0.005$ gm./c. cm. (estimated)
$\Delta\rho=0.00005$ gm./c. cm. (given)
$\Delta r=0.0001$ cm. (estimated)
$\Delta l=0.05$ cm. (estimated)

Putting these deta into the above formula, we obtain the relative mean error of viscosity measurement : $\frac{\Delta\eta}{\eta} = \pm 0.8\,\%$.

Finally, our result are led to :

$$\eta = (6.28 \pm 0.05)\ \mathrm{P}, \quad 0.8\,\%\ \text{at}\ 29\,^\circ\mathrm{C}.$$

IV. STROBOSCOPIC EXPERIMENT

In oder to examine the initial transitional phenomena of falling spheres through liquid until a terminal velocity is reached, we use the rotating shutter strobe camera[2)~5)] shown in Fig. 3 was set up.

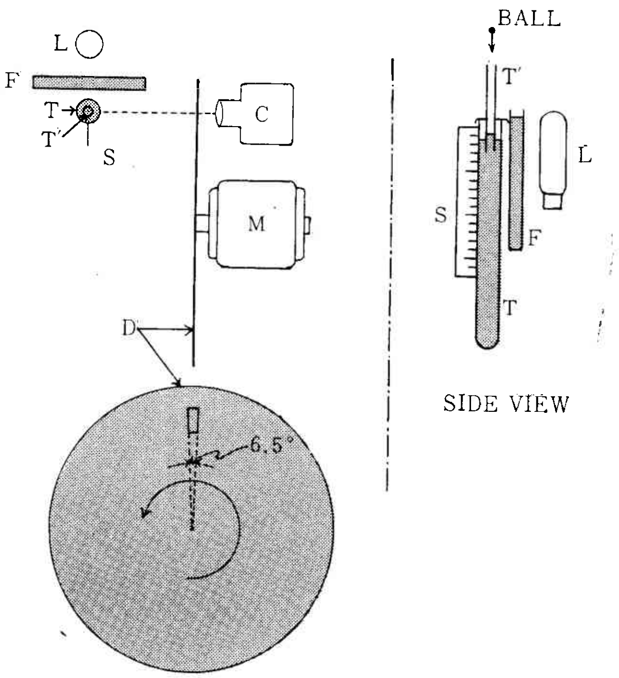

Fig. 3 Schematic diagram of the experimental arrangement.

In this figure D is a metal disk with one slit, which is rotated at exactly 1800 rpm by a synchronous motor driven at 60 cps alternating power. T is the vessel filled with glycerine. A piece of glass tube T′ is inserted in the vessel. By allowing ball to fall down through this tube, it may be dropped centrally in the vessel. As a source of illumination the high pressure mercury lamp (50 W) was used. To avoid the heating effect by the lamp a water vessel F is placed as a filter. The scale S is attached to the tube. The camera (lens f=50 mms, F : 3.5) is mounted at a distance about 18 cms. from the object. Through a slit of the rotating disk the photograph of a successive position of falling sphere can be taken at intervals 1/30 sec.

The photo of falling spheres is shown in Eig. 4. From this the actual distances of falling sphere at intervals 1/30 sec. may be calculated by using a measuring microscope. We have numbered these distances ; the position in the vicinity of lower end of tube T′ is taken as initial numbering point. The plotting of distances vs the number is shown

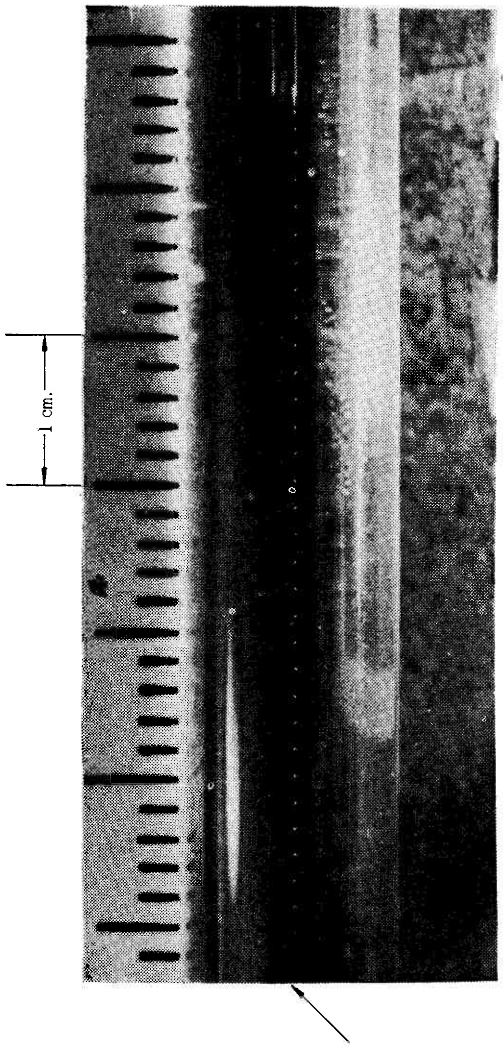

Fig. 4 Actual photo of falling balls.

in Fig. 5.

From the graph (shown in Fig. 5) the apparent terminal velocity is given as $v =$ 5. 70 mms. /sec., which is in very good agreement with the result obtained by another falling experiment illustvated on page 1. With careful work an over-all erorr of 0. 2 % is possible.

V. CONCLUSION

Our experiments provide a test of several consequences of Eq. (2) and the perfect agreement is shown. Accuracies of our experimental results are generally quite satisfactory in our student laboratory. There are many reasons to use the Stokes's law in the experiment. Its simplicity and lucidity are attractive ; and as it was used to solve fundamental problems in physics by Einstein, Perrin and Millikan, it also may lead the students to a better appreciation of the basic principles of physics.

The strobe camera apparatus allows the students to visualize the transitional pheno-

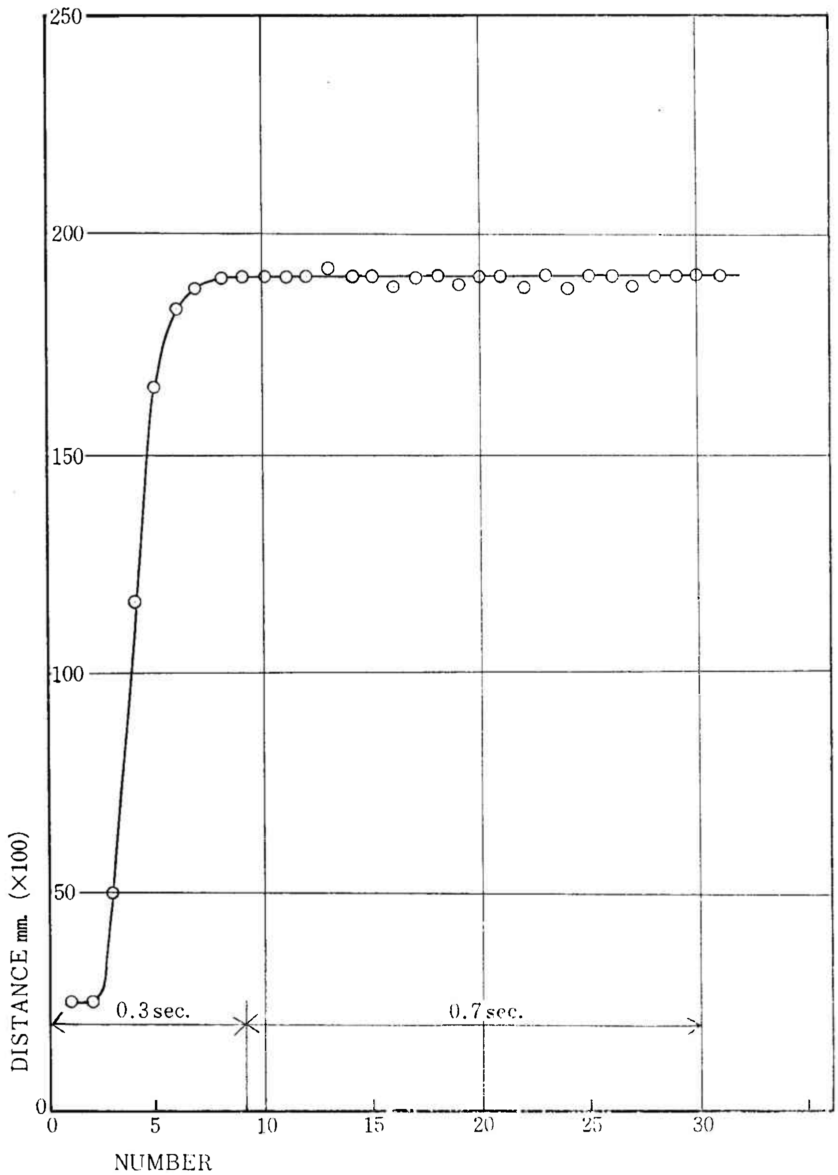

Fig. 5 The plot of distances of successive intervals.

mena. As shown in Fig. 5 some fluctuatios from the stational state in the later stage of fall can be observed. It may be due to oscillations of the sphere during its decent in a stational liquid column. Further experimental details and results will be presented in a later publication.

ACKNOWLEDGMENTS

The author would like to take this opportunity to thank Chief Director K. Fujii for his constant encouragement. Many thanks are also due to President Professor N. Nakamura for his kind guidance. The author is thankful to many students who have assisted us in this experiment and who have served an unwitting guninea pigs in helping us to learn "Das Lernen des Lernens".

REFERENCES

1) Westphal, physik, §78 ; Kl. Lehrb. d. Physik, §70.
2) D. L. Enlow and P. A. Schroeder, Am. J. Phys. 35, 651 (1967).
3) E. Hurrins, Physics I (W. A. Benjamin Company, Inc., N. Y., (1965).
4) W. E. Hazen et al., Am. J. Phys. 27, 174 (1959).
5) W. W. McCormick, Laboratory Experiments in Physics (MaciMllan Co., N. Y., 1966).